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THE INDEX CYCLE: A CROSS-SPECTRAL ANALYSIS OF ZONAL INDEX DATA

PAUL R. JULIAN

National Center for Atmospheric Research, Boulder, Colo.

ABSTRACT

In order to examine quantitatively the evidence for an index cycle, 21 years of daily 700-mb. zonal index data for the western longitudes of the Northern Hemisphere were subjected to cross-spectral analyses. The annual variation of the mean and standard deviation of the three zonal indices (polar, temperate, and subtropical) was removed by a standardizing procedure utilizing time-smoothed annual curves.

The variance (power) spectra of the individual indices indicated that their correlation structure can be closely approximated by a first-order Markov scheme. No evidence was found for an excess of variance in the range of period (3 to 8 weeks) assigned to the index cycle by subjective means. The cospectra all indicated negative covariability among the indices as would be expected from circulation patterns characteristic of index-cycle-like behavior, but none showed an excess of negative covariability within the period range of interest. Coherence values, indicating the degree of relationship among the various indices, also did not show maxima in the required period range.

The distributions of the standardized indices were found to be slightly skewed in the case of the subtropical and temperate indices and normal in the case of the polar index. In addition, a significant seasonal variation in the correlation and cross-correlation structure of the indices was found. Such nonstationarity complicates the interpretation of the cross spectra.

The conclusion of the statistical study of the zonal index data is that there is no evidence for sequential variations and covariations of the strength of the westerlies at various latitudes with any preferred range of period. Rather, significant relationships hold over a very broad range of period.

1. INTRODUCTION

Petterssen [15] describes the concept of the index cycle as being founded on the pioneering work of Willett, Rossby, and Namias on the development of circulation patterns in the westerlies. He defines the phenomenon as ". . . irregular quasi-cyclic changes [in the basic zonal circulation] the period of which may vary from about 3 to 8 weeks." Namias and Clapp [12] summarize the apparent tendency of the zonal westerlies to oscillate between high index values representing a strong zonal flow and low index characterized by strong meridional flow and considerable meandering of the belts of westerlies. As the decrease from high to low index is taking place there is also a tendency for the main belt of westerlies to be displaced equatorward. Willett and Sanders [17] give a description of the index cycle which involves the sequential occurrence of three types of circulation patterns characterized by high or low zonal index conditions. They also state that the period range for the length of the index cycle is from 3 to 8 weeks.

There is, apparently, a question as to how often this quasi-cyclical variation takes place. Namias and Clapp emphasize a "primary index cycle" occurring normally just once each winter, whereas Petterssen states that the variations in the zonal index are simply much larger in winter than in summer. Willett and Sanders (p. 195) discuss the seasonal variations in the characteristics of the index cycle maintaining that the phase is "sharpest" (?) in autumn and weakest in late winter and spring. In any case the statement of Namias and Clapp (p. 561) that ". . . its precise form of operation, time of onset, and even its length and intensity are subject to appreciable variations," indicates the somewhat tenuous nature of the index cycle. Considering the caution of the statement, and considering the fact that no quantitative demonstration of the existence of an index cycle has been made, it is surprising to find how commonly it is referred to in the meteorological literature.

The phenomenon of vacillation in the dishpan experiments [5] is thought, moreover, to be pertinent to the problem of atmospheric index cycles. In the laboratory the range of frequency of vacillation in the annulus experiments is more restricted than that assigned to the atmosphere, while in the open-center apparatus few data have been taken which could be contrasted with atmospheric zonal indices.[1]

However, if a phenomenon exists which is singular enough to require description in terms of the regularity of circulation patterns and which suggests a comparison with the vacillation phenomenon of the dishpan experiments (Petterssen, p. 289), then one would expect it to be detectable above the noise of ordinary meteorological variability.

There are two features of the index cycle as described by the above authors that suggest the use of a statistical analysis of zonal index data. The first is the restriction of the frequency of the index cycle within a limited *range*. There is certainly the implication in the work cited above [17], [15], [12], that oscillations in the zonal index with periods from 3 to 8 weeks should be more pronounced than oscillations with longer or shorter periods. The second feature of the index cycle capable of statistical testing is the latitudinal shift in the main belt of westerlies. Such a shift should produce a negative covariance between subtropical and middle or high latitude zonal indices, and further, this negative covariance should be greatest for the range of frequencies characteristic of the index cycle itself.

2. THE TECHNIQUE OF CROSS-SPECTRUM ANALYSIS

Cross-spectrum analysis is ideally suited for a study of the index cycle because it expresses the distribution of covariance of the strength of the zonal current at two different latitudes as a function of the frequency of oscillation. Thus, if there is a tendency for the expansion and contraction of the main zonal current with its associated meanders to occur with a preferred range of period, the cospectrum of zonal wind intensity at two latitudes sufficiently far apart should exhibit a minimum at the preferred range of period or frequency. Moreover, if interest is focused upon the degree of association between the zonal westerlies at two latitudes, the coherence statistic should indicate that oscillations with the preferred period are better related than oscillations with other periods.

Panofsky [13] performed such an analysis using 500-mb. hemispheric zonal index data for two periods, each about 3 years in length, although he was not concerned with the index cycle phenomenon. Because of the limited period covered by his data and of certain limitations in their basic form, it seems warranted to repeat the analysis. Here, as in Panofsky's study, the complete cross spectrum is calculated. For a discussion of the interpretation of the cospectrum, quadrature spectrum, and coherence the reader is advised to consult standard texts (e.g., Panofsky and Brier [14], Lumley and Panofsky [10], Lee [9]).

It is appropriate here to note, however, that some confusion exists in the literature as to the meaning of the term "coherency" or "coherence." Most statisticians define the coherence as follows:

$$R_{xy}^2(f)=\frac{C_{xy}^2(f)+Q_{xy}^2(f)}{S_{xx}(f)S_{yy}(f)} \tag{1}$$

Where C_{xy} (f) is the cospectrum of series x and y at frequency f, $Q_{xy}(f)$ the quadrature spectrum, and S_{xx} and S_{yy} the power (variance) spectra of the x and y series, respectively. Reference to the coherence then usually, but not always, means to R_{xy}^2 and not to R_{xy}. Because of the analogy to a correlation coefficient, it seems natural to refer to R_{xy} as the coherence. To indicate this inclination and to prevent confusion, Haubrich [4] uses the term "squared coherence" for R_{xy}^2. In agreement with Haubrich all reference in this paper will be to R_{xy}, the square root of expression (1).

3. DATA

The raw data, kindly supplied by the Extended Forecast Branch of the National Meteorological Center, consist of 21 years (1944–64) of daily 700-mb. zonal index (hereafter Z.I.) values. Table 1 indicates the latitude and longitude intervals used in the computation of each index.

The daily data are in meters per second. Because of the standardization procedure described in the next section the data subjected to the cross-spectrum analysis were dimensionless numbers.

The total number of data values for the 21-yr. period is 7671. To check certain features appearing in the cross spectrum computed from the full set, the data were divided into two discrete sets of 3835 and 3836 values, and cross spectra computed for each. Table 2 lists the relevant parameters for the various cross-spectra runs.

The cross spectra were computed by the standard Tukey [1] technique using the "hanning" smoothing procedure.

TABLE 1.—*Definitions of zonal indices used in study*

Name	Latitude Belt °N.	Longitudes included
Subtropical	20	0° westward to 180°.
	35	5° W. to 175° E.
Temperate	35	5° W. to 175° E.
	55	5° W. to 175° E.
Polar	55	5° W. to 175° E.*
	70	0° to 180°.

*From 1944–50 this was 0–180.

[1] Private discussion with D. Fultz, University of Chicago.

TABLE 2.—*Data summary of cross-spectral analysis*

Run no.	Number of data, N	Time interval	Total no. of lags, M	Frequency resolution (cycles/day)	Degrees of freedom (after [1])	$\frac{4N}{3M}$
1	7671	Jan. 1, 1944 to Dec. 31, 1964.	500	10^{-3}	30	20
2	3835	Jan. 1, 1944 to July 1, 1954.	100	$5\cdot10^{-3}$	76	51
3	3836	July 2, 1954 to Dec. 31, 1964.	100	$5\cdot10^{-3}$	76	51

4. STATIONARITY AND THE REMOVAL OF THE ANNUAL VARIATION

To obtain the maximum benefit from the estimation procedure and for simplicity of interpretation of the spectra, time series subjected to cross-spectrum analysis should be stationary. Meteorological time series are almost never stationary, primarily because of the pronounced diurnal and annual variations of the physical processes generating the variables. Occasionally such time series may be made stationary, at least in some degree, by the removal of the nonstationary variation—in the case of daily zonal index values by removal of the annual variations.

For this purpose it is assumed that the annual variation in the various zonal indices is a deterministic function of arbitrary form in the first two moments of the distribution. That is, the mean and standard deviation of each index undergo an annual oscillation that is not necessarily the sum of any finite number of ordinary (Fourier) harmonics. The daily indices are then standardized by removal of the appropriate mean for the day of the year and by division by the appropriate standard deviation.

To perform the standardization by an arbitrary annual wave shape, rather than by a harmonic analysis, the following procedure was adopted. Data for each of the 52 weeks of the year for the 21 yr. of data were averaged:

$$MZ(I)=\sum_{J=1}^{21}\sum_{K=7I-6}^{7I} Z(J,K),$$

J=year index; K=day index; I=week index

and

$$SDZ(I)^2=\sum_{J=1}^{21}\sum_{K=7I-6}^{7I}[Z(J,K)-MZ(I)]^2$$

with the 365th and 366th days neglected. To produce a smooth curve of the mean and standard deviation the values resulting from the grouping above were subjected to a curve-smoothing procedure known as double integration. This procedure, which is appropriate for sinusoidal curves, has been described, e.g., by Langbein [8], and has the attractive property that a smooth curve of arbitrary shape results. Two successive series of progressive totals of the unsmoothed values are formed. Thus if the data approximate the form

$$y\approx a+b\sin\frac{2\pi t}{P}\pm\epsilon$$

in which P is the period of the sinusoidal component and ϵ represents a residual (a and b are arbitrary constants) the second integration gives

$$y-a\approx -b\left(\frac{P}{2\pi}\right)^2\sin\frac{2\pi t}{P}\pm\sum\sum\epsilon$$

with the residual term now being much smaller than in the original series.

A plot of the smoothed values of the week's mean and standard deviation for the three Z.I.'s is given in figure 1. To standardize the daily indices, the weekly values were specified to occur in midweek and the daily mean and standard deviation calculated by simple linear interpolation. It should be evident from the figure that the unsteadiness introduced by this interpolation procedure is small and well within the original uncertainty of the zonal index data.

The procedure followed here is intended to relieve the original data of the annual oscillation in the mean and variance. Nonstationarity in the higher moments of the distribution or in the conditional probability distribution governing the Z.I. series may still exist. Material in sections 9 and 10 pertains to this problem.

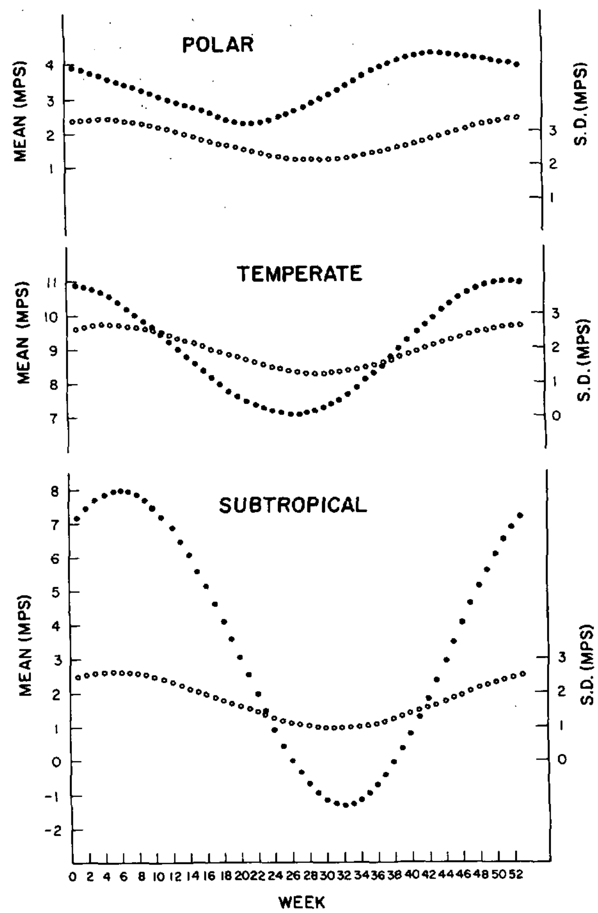

FIGURE 1.—The seasonal variation of the mean (solid circles) and standard deviation (open circles) of the three zonal indices. Units are meters per second. The curves give the result of the smoothing process described in section 4.

5. REMARKS ON THE REPRESENTATION AND INTERPRETATION OF CROSS SPECTRA

The figures showing the variance spectra and cospectra of the three Z.I. series possess a logarithmic rather than a linear scale of frequency. The advantage of this method of presentation is obvious when interest is in a considerable range of period or frequency. In order to preserve the desirable feature of a spectral plot in which area is proportional to variance, the spectral estimates are multiplied by the frequency (see Lumley and Panofsky [10]).

Because spectrum analysis results in a decomposition in linear frequency bands, and because a purely random or white noise spectrum is by definition a constant independent of frequency, care must be exercised in interpreting spectra given in semilogarithmic coordinates. A white noise spectrum, $S(f)$=constant, when plotted in such coordinates with the area-variance relationship preserved would no longer be a constant since $fS(f)$ would be the ordinate. This representation would have a maximum at 0.5 cycles per sampling interval (Nyquist frequency). A Markov process spectrum, which on a linear frequency scale has a maximum spectral density at $f=0$ and decreases *uniformly* to a minimum at the Nyquist frequency, possesses a hump or maximum when plotted in the semilogarithmic coordinates (e.g., figs. 2–4). One would scarcely say that a time series with a red-noise [2] or simple persistence structure possessed a quasi-periodicity, yet one might be tempted to do so by interpreting a Markov spectrum in semilogarithmic coordinates. In figure 2, for example, the eye is attracted to the "peak" at the 30-day period. The proper interpretation of the spectra must be based upon recognition of the difference between the spectral continuum and peaks which represent actual quasi-periodicities.

SUBTROPICAL Z.I. VARIANCE SPECTRUM

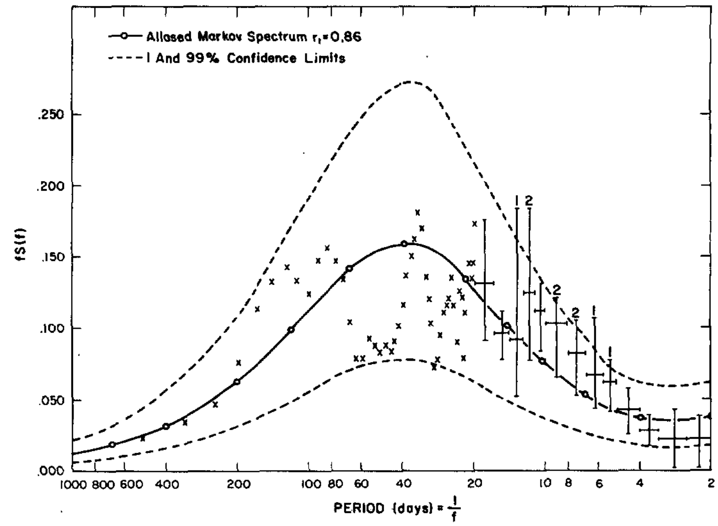

FIGURE 2.—Variance spectrum of the standardized subtropical zonal index, January 1944 to December 1965 (Run No. 1, table 2). The abscissa is a logarithmic period scale in days and the ordinate is $fS(f)$ in standardized units. The solid and dashed curves are the theoretical, aliased Markov spectrum and its 1 percent and 99 percent sampling limits giving the best fit to the sampled Z.I. spectrum. Spectral estimates for periods less than 20 days are averaged over discrete ranges of period as indicated by the extent of the horizontal lines. The greatest and least estimate in each period range is indicated by the length of the vertical lines. Numbers above the 99 percent sampling curve indicate the number of estimates exceeding that limit.

6. RESULTS OF THE SPECTRAL ANALYSES

In figures 2–4 the spectral estimates are plotted as single points for periods exceeding 20 days (or 3 weeks). For shorter periods adjacent estimates are averaged in discrete blocks, with the range of period covered and the maximum and minimum spectra estimated therein indicated.

Because the variance spectra of all three indices are characteristic of "red-noise," the appropriate spectrum for a simple Markov process has been fitted by eye to the estimates in each figure. As indicated in the legends, the aliased Markov spectrum and its 1 percent and 99 percent limits following the modification of the Tukey sampling theory suggested by Gilman et al. [2] are shown. Discussion of the sampling problem is deferred to section 8.

Apparently, over a very considerable range of frequency the individual zonal indices possess a spectrum given by simple persistence; that is, by a first-order Markov process. In the aliased Markov spectra shown, variance from periods less than 2 days is aliased into the range of frequency covered by the analysis, resulting in the slight upturn of the spectra at 2–3 day periods. The data, however, do not exhibit this upturn; and, in fact, inspection of the actual data points indicates that in the range of 20 to 2 days the variance spectra are falling off more rapidly with decreasing period than a Markov spectrum.

More important in figures 2–4, however, is the lack of any range of period subjectively assigned to the index cycle containing significantly more variance than the background red-noise variance. Outside of the 2–5-day period range, no more spectral estimates exceed the 1 percent limits than expected by chance, and only two exceed these limits in the range 3 to 8 weeks.

Monin [11], has reported some work done with a 600-mb. mid-latitude zonal index, presumably covering some portion of the Eastern Hemisphere, which indicates spectral peaks at 12 and 24 days. Although he did not attempt significance tests of these peaks, he associated the 24-day oscillation with the index cycle. These two peaks have questionable counterparts in figure 3 where two estimates with periods of about 21 or 22 days exceed the 1 percent sampling limit, and at 12–13 days where three estimates exceed the limit. When the temperate Z.I. was divided into two halves (runs 2 and 3, table 2) both showed peaks

[2] A term suggested by E. N. Lorenz for spectra having a major portion of the variance in the lowest frequencies when presented with a linear frequency scale.

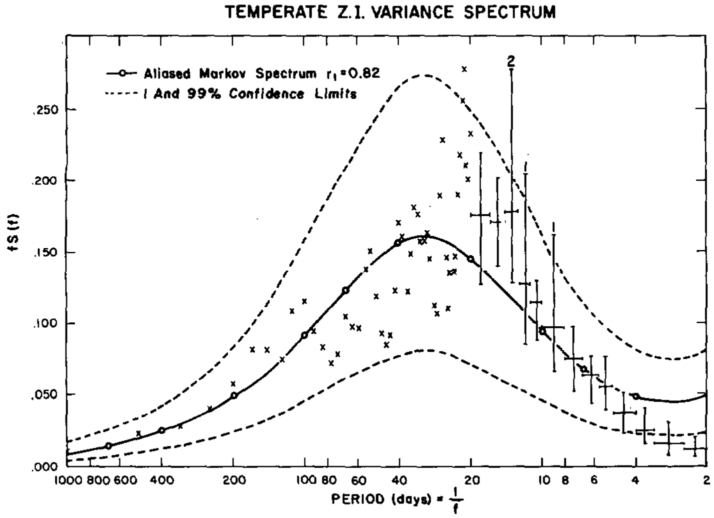

FIGURE 3.—Same as figure 2 but for the temperate zonal index.

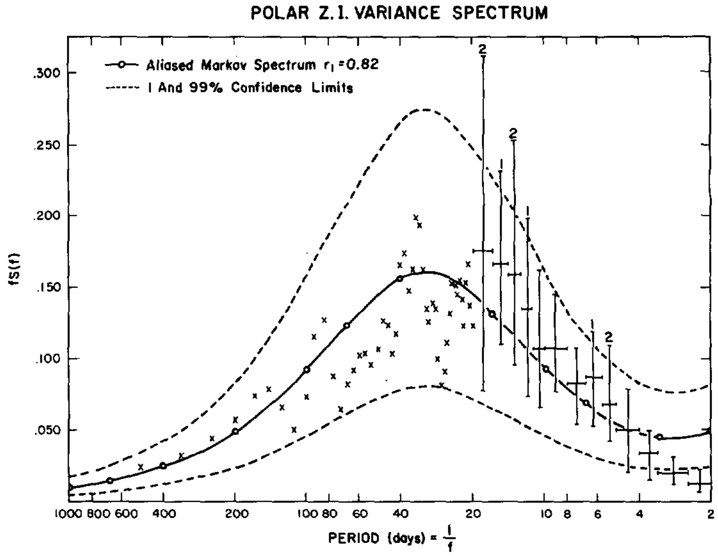

FIGURE 4.—Same as figure 2 but for the polar zonal index.

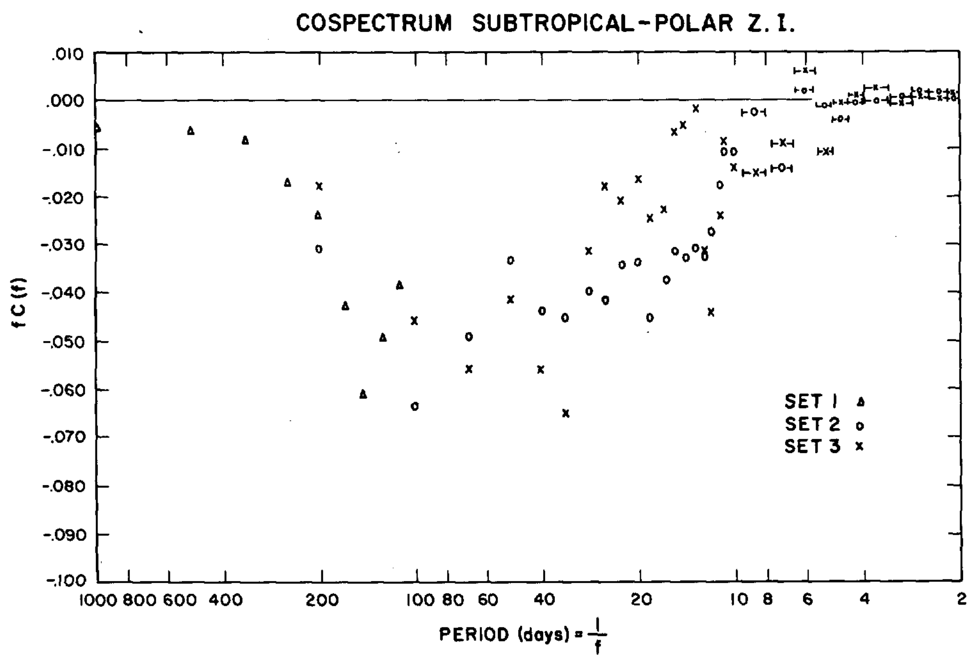

FIGURE 5.—Cospectrum of standardized subtropical-polar zonal indices. Data from all three runs (table 2) are included. The abscissa is a logarithmic period scale in days and the ordinate is $fS(f)$ in standardized units. Cospectral estimates for periods less than 10 days are averaged over discrete ranges of period and are presented as in figures 2 to 4.

at approximately the same frequencies,[3] but both peaks were of doubtful significance. However, significant or not, neither of these peaks is broad enough to qualify as the manifestation of an index cycle.

From the individual variance spectra, then, there appears to be no evidence of oscillations of the individual Z.I. with a preferred range of period.

[3] The fact that the peaks were not at the same frequencies in the two halves in itself could detract from their significance.

7. RESULTS OF THE CROSS-SPECTRUM ANALYSIS

The cospectra, showing the distribution with frequency (period) of the in-phase covariation at the Z.I.'s, are given in figures 5–7. All cospectra are negative at all periods resolved except for a range of period about 100 days for the polar-temperate combination (fig. 7), and for the very short periods, less than 6 days, in the subtropical-polar cospectrum (fig. 5). The maximum negative value in the cospectral continuum appears to be reached in the

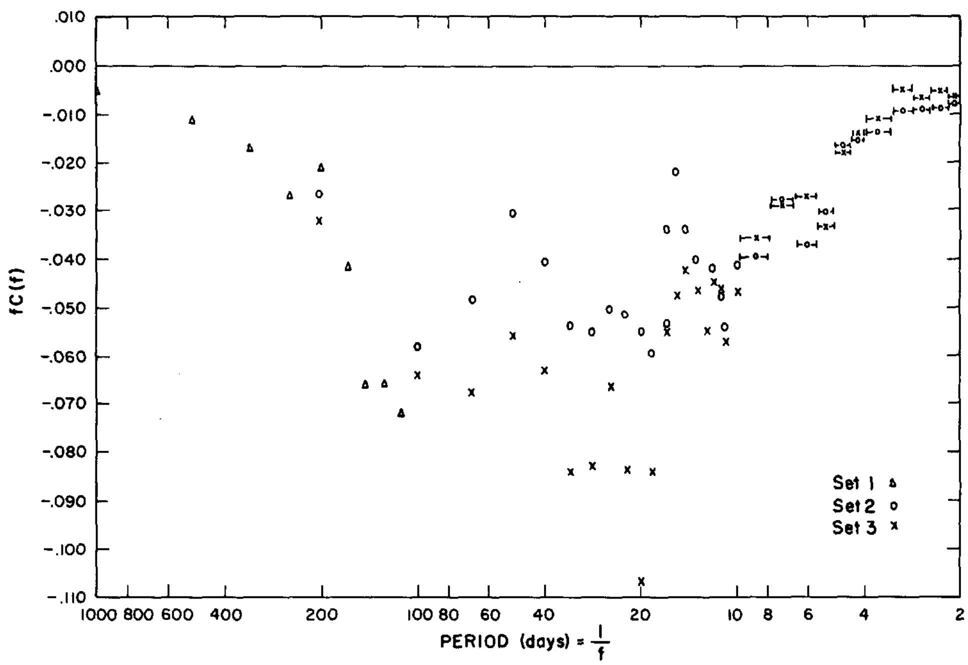

FIGURE 6.—Same as figure 5 but for subtropical-temperate pair.

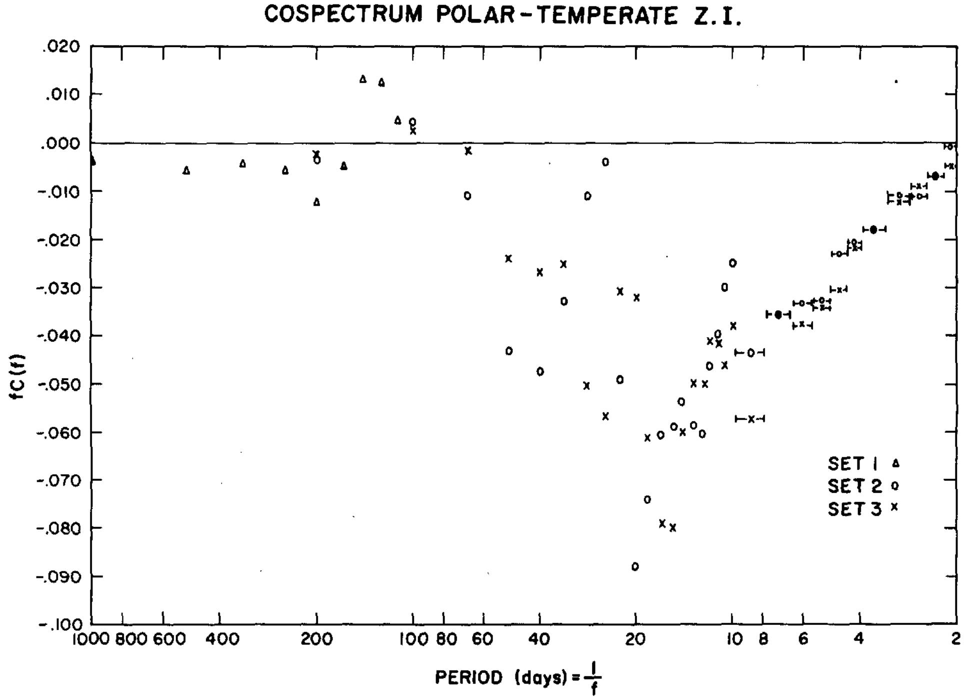

FIGURE 7.—Same as figure 5 but for polar-temperate pair.

range of period 20 to 40 days, except for the subtropical-polar pair. Compared with the other two cospectra this minimum occurs at longer periods, about 40 to 100 days.

The strong negative covariance between all three indices at periods of 2 to 14 weeks is a reflection of the tendency for the zonal circulation to shift from one latitude belt to another. The 3- to 8-week period range does not stand out above the background spectrum: this situation is quite analogous to that of the individual variance spectra. The broad maxima in the cospectra should not, therefore, be interpreted as evidence for a quasi-cyclical covariation, just as a Markov spectrum cannot be considered quasi-cyclical.

The quadrature spectra (not shown) are generally an order of magnitude less than the cospectra. The only quadrature spectral estimates that suggest a non-zero population spectrum are for periods longer than about 25 days for the subtropical-temperate combination. These estimates are negative and average from about half to about the equal of the cospectral values. These negative spectral estimates, if significant, together with the negative cospectra at the same frequencies indicate a lag of the subtropical index (in this case) behind the temperate.

Figures 8–13 show the coherence statistic, R, as a function of the lag, l, or the frequency, on a linear scale. A period scale in days is appended at the top of each figure. Figures 8–10 are from runs 2 and 3 and the coherence values from both runs are plotted together for comparison. To obtain a better resolution in the period range of interest, the coherence from run 1 (all 21 yr. of data) is given in figures 11–13 for the first 50 lags only, corresponding to periods longer than 20 days.

Figures 8 and 11, presenting the coherence between the subtropical and the polar Z.I., are of primary importance. One might argue that if the features of the index cycle described by Namias and Clapp are indeed characteristic of a particular range of period, the coherence between the polar and the subtropical westerlies should be greatest in that range of period. Since both individual series possess maximum variance in the same range of period wherein the covariance is a maximum, perhaps it is the analog of the correlation coefficient, the coherence, which is important.

In figure 8, it is obvious that it is only for oscillations with periods greater than 20 days that any reasonably large value of the coherence is reached at all; and in figure 11 it is obvious that the period range 3 to 8 weeks (about 20–60 days) is not especially favored by outstanding values of R. The few peak coherence values at slightly less than 100 days and at about 35 days do not occur over a wide enough frequency range to suggest an index cycle and in a statistical sense probably do not represent significant peaks above the background continuum. (See next section.)

The remainder of the figures showing the coherence of the Z.I.'s in adjacent latitude belts also do not give the impression of a favored range of period. They do indicate that the subtropical and temperate indices are equally related at all frequencies resolved, and that the polar and temperate indices are best related for periods less than about 10 days.

Because of the presence of a common boundary, the coherence values of zonal index fluctuations for adjacent latitude belts contain an automatic non-zero component. Except to note that it is negative it is not worthwhile in the context of the present article to attempt to ascertain the magnitude of this automatic correlation. However, whatever the magnitude might be, it should not be a function of frequency. Therefore, a variation in the

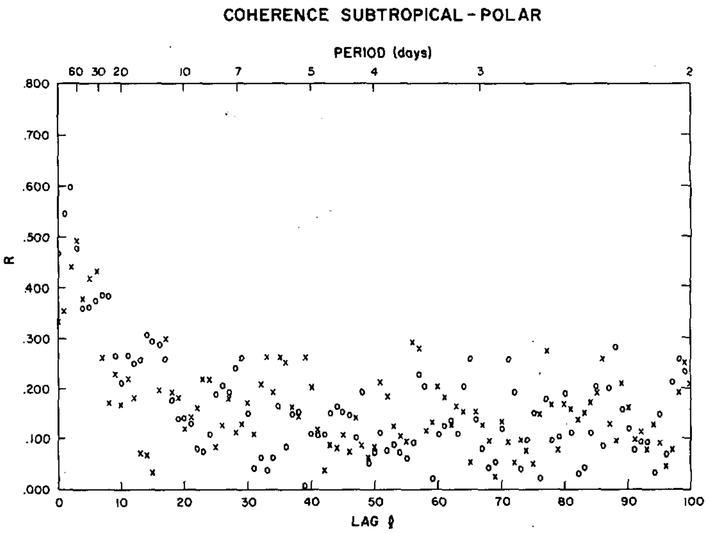

FIGURE 8.—Coherence (square root of expression (1)) for subtropical-polar Z.I. pair (Runs No. 2 and 3, table 2). A period scale in days is given at the top of the diagram.

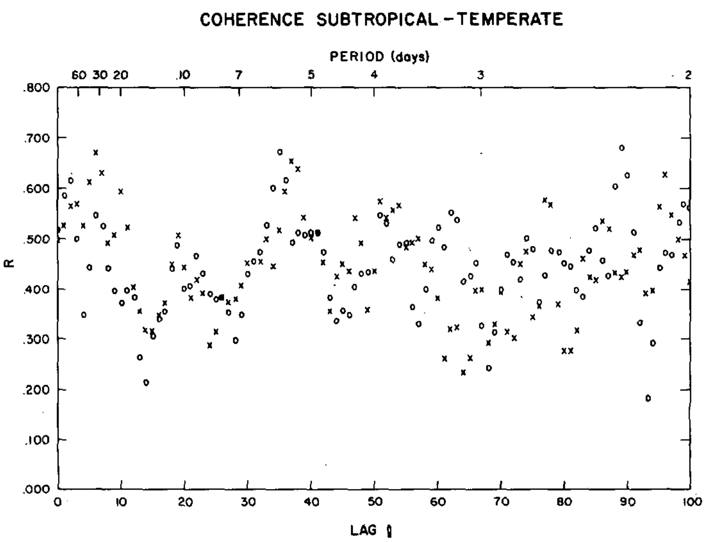

FIGURE 9.—Same as figure 8 but for subtropical-temperate pair.

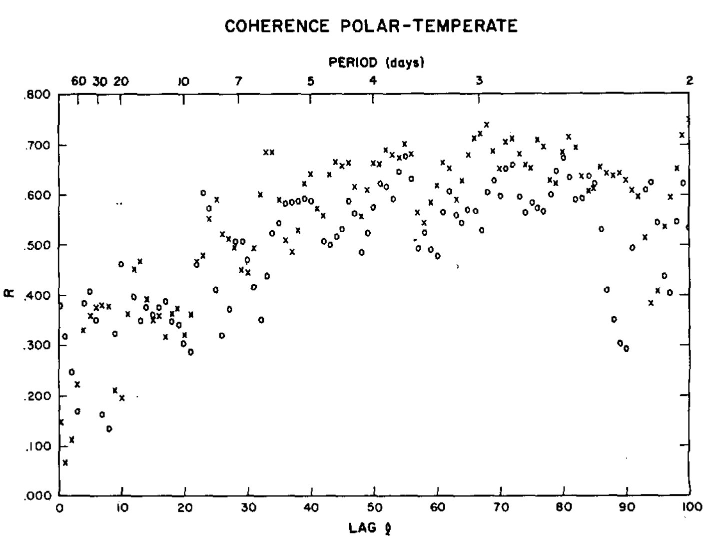

FIGURE 10.—Same as figure 8 but for polar-temperate pair.

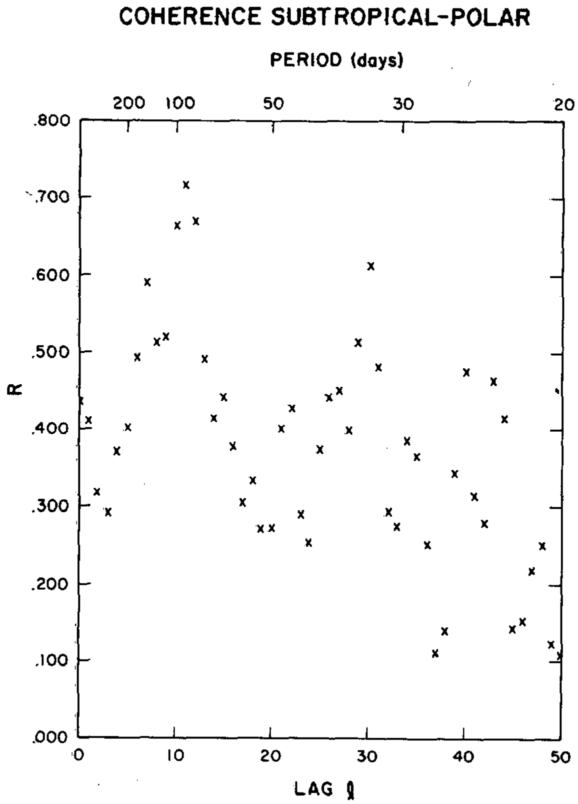

FIGURE 11.—Coherence (square root of expression (1)) for subtropical-polar Z.I. pair (Run No. 1, table 2) for frequency range 0 to 0.05 per day. A period scale in days is given at the top of the diagram.

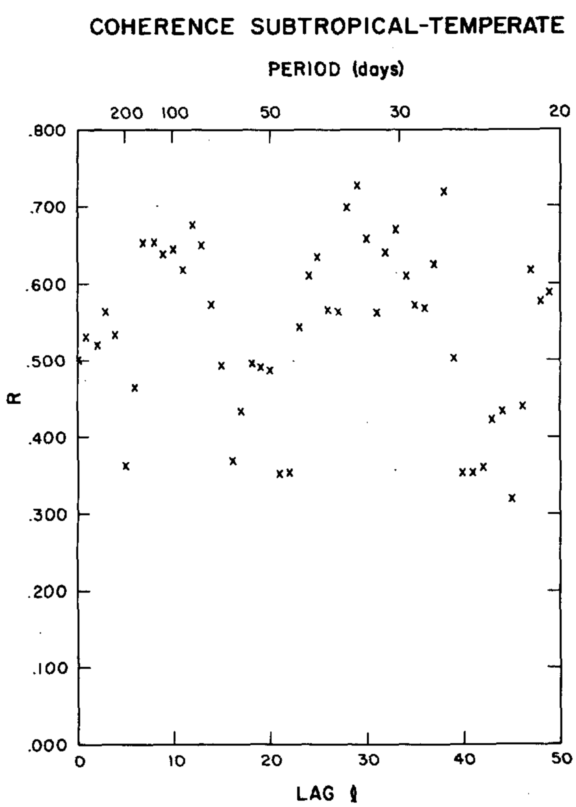

FIGURE 12.—Same as figure 11 but for subtropical-temperate pair.

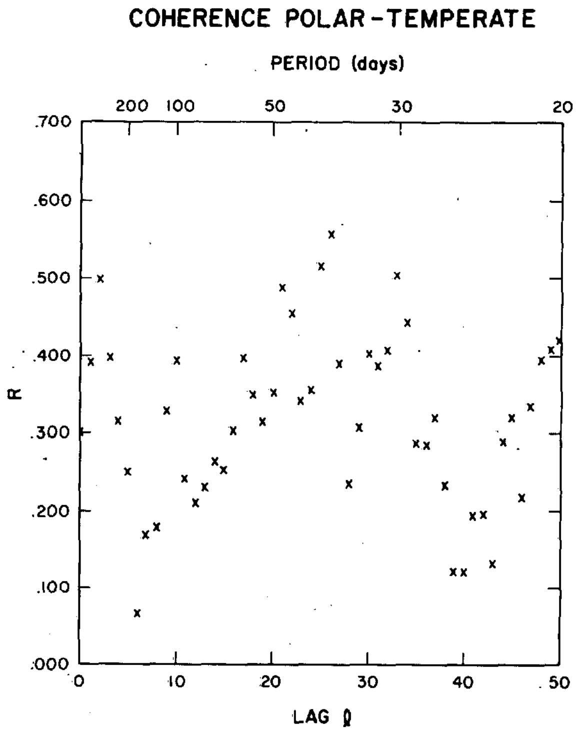

FIGURE 13.—Same as figure 11 but for polar-temperate pair.

coherence statistic is indicative of a non-random behavior of the covariance of (longitudinally averaged) pressure in the meridional direction. Although such a non-random variation might not be the manifestation of an index cycle, its existence is of some interest in studies of the general circulation.

In the coherence diagrams for the two pairs of adjacent zonal indices, figures 9 and 10, a contrast in behavior is readily apparent. For the subtropical-temperate pair the coherence values scatter about what very probably is a constant population value, whereas for the polar-temperate pair the population coherence increases from low values at the lower frequencies to high values at the high frequencies. In the subtropical latitudes the covariance of pressure in the meridional direction is, therefore, random in time; but in the more northerly latitudes a significant non-random behavior of the covariance of pressure exists. Here the covariance of pressure in the meridional direction for the lower frequency oscillations (periods greater than about a week) is quite different than that for the higher frequency oscillations (periods less than a week). Tentatively, this non-random behavior in the higher latitudes may be assigned to the phenomenon of blocking, as described, e.g., by Rex [16].

8. SAMPLING THEORY AND THE SIGNIFICANCE OF THE SPECTRAL ESTIMATES

A sampling theory of cross-spectral estimates has been attempted by Goodman [3] which has been generalized in a more easily obtainable paper by Jenkins [7]. All sampling theories proposed to date, including that for variance spectra estimates, are based upon random sampling theory. They give as a result a quantitative expression for the variability of random deviations from the population spectrum at a given frequency. There are several features of such theories which often make their application in geophysics very difficult. The first is that the population value is in general not known. For example, the Tukey theory states that the spectral estimates are distributed about the population spectrum as chi-squared upon the number of degrees of freedom. With but a single realization of a time series in hand it is virtually impossible to interpret the resulting sample spectrum in terms of large, random sample theory.

Furthermore, as discussed by Jenkins [6], the theory does not consider the covariability of estimates at different frequencies. Thus the sampling theory does not apply to hypotheses concerning the background spectrum arising because of the general auto- or cross-correlation structure of the time series.

Gilman et al. [2] have suggested a possible procedure in the case in which it is desired to test the significance of a peak above a background spectrum given by simple persistence. Their procedure has been followed in figures 2–4. However, the significance limits derived by this approach must certainly be too conservative because of the uncertainty in choosing the background spectrum. Moreover, examination of nearly adjacent spectral estimates in many red-noise spectra (e.g., those in figs. 2–4) indicates that they are not independent of each other. Groups of contiguous estimates tend to fall either above or below the assumed background spectrum, thus vitiating the testing of peaks comprised of more than two or three estimates using the Tukey-Blackman theory.

The foregoing discussion makes it evident that no satisfactory sampling theory applicable to the problem of the index cycle exists. A procedure similar to that of Gilman et al. would be very difficult to apply. First, a model describing the covariance structure of two persistent series would be needed to specify the background spectrum. Secondly, the range of period or frequency band of interest is becoming a significant portion of the total range of period resolved by the analysis, thus obscuring the difference between the frequency band of interest and the background spectrum.

Neglecting the difficulties with the present sampling theory we may attempt to test the significance of the peak coherence values in figure 11. The results of the theory of Goodman have been given in convenient graphical form by Haubrich [4]. We assume a continuum of population coherence, R_∞, which decreases with increasing frequency: for the peak at 90 days R_∞ is about 0.42 and for the peak at 35 days, about 0.35. With 20 ($4N/3M$) degrees of freedom the upper sampling limits (0.95 limits) would be 0.76 and 0.72 respectively. Thus neither peak is significant at the 5 percent level. Moreover, periods in the range of about 90 days are somewhat outside the range assigned to the index cycle by subjective means. Examination of figure 8 indicates that this peak is characteristic of only half of the total 21-yr. record. For these reasons the conclusion that these coherence values do not indicate statistical evidence for an index cycle is justified.

The conclusions reached as a result of the cross-spectral analysis are: (1) No indication is available that oscillations in the zonal index data conform to what would be expected on the basis of the synoptic description of the index cycle. Rather, the significant covariability of two zonal indices is spread out over a relatively broad range in period. (2) By taking into account the fact that each of the Z.I.'s individually has maximum variability in the same broad range in period, the coherence statistic indicates that periods of from 3 to 8 weeks are not better related than the longer periods in the case of the subtropical-polar Z.I. combination. On the other hand it is periods shorter than about 2 weeks which show the strongest relationship of the polar and temperate Z.I.'s. In the case of the subtropical and temperate indices, all periods resolved are equally well related.

9. DISTRIBUTION OF STANDARDIZED INDICES

It is of some importance to investigate the statistical distribution of the standardized index values. The efficiency of the cross-spectrum estimation and of any application of sampling theory are dependent upon the degree of normality of the variates, although it does not seem to be theoretically possible to state quantitatively an exact dependence.

The 7671 values of each zonal index were divided into 10 classes, the class limits of which were determined by imposing an equal probability of occurrence in each class according to the Gaussian (normal) distribution. Table 3 gives the chi-square analyses based upon the observed and expected occurrences in each class interval.

The polar index is the only one of the three that is normally distributed at the 5 percent level of significance. Both subtropical and temperate indices are skewed, interestingly enough one positively and the other negatively so. Since the seasonal variation in these latter indices is so much greater than in the polar index one might be inclined to attribute the skewness to the standardization procedure. However, as the sign of the skewness is different in the two cases whereas the seasonal trends are similar, it is difficult to accept such a conclusion.

In spite of the skewness of the temperate and subtropical indices it is apparent that neither distribution is highly abnormal, and that therefore the spectral estimation and sampling distribution techniques are very probably valid ones.

The possibility exists that the skewness is present only during a particular season or seasons of the year, preventing stationarity of the third moment in the distribution. It was felt, however, that the magnitude of the skewness was not sufficient to warrant an investigation.

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TABLE 3.—*Chi-square test of normality of Z.I. frequency distribution*

Class	Observed frequency	Differences	χ^2
SUBTROPICAL Z.I.			
Expected frequency 767.1			
I	694	−73.1	6.96
II	846	+78.9	8.12
III	878	+110.9	16.04
IV	826	+58.9	4.52
V(−)	728	−39.1	1.99
VI(+)	759	−8.1	0.08
VII	688	−79.1	8.16
VIII	705	−62.1	5.03
IX	733	−34.1	1.52
X	814	+46.9	2.87
			55.29
χ^2 for $P=0.05$ with 9 degrees of freedom=16.92			
TEMPERATE			
Expected frequency 767.1			
I	823	+55.9	4.07
II	727	−40.1	2.10
III	734	−33.1	1.43
IV	686	−81.1	8.57
V(−)	715	−52.1	3.54
VI(+)	782	+14.9	0.29
VII	770	+2.9	0.01
VIII	861	+93.9	11.49
IX	833	+65.9	5.66
X	740	−27.1	0.96
			38.12
χ^2, $P=0.05=16.92$			
POLAR Z.I.			
Expected frequency 767.1			
I	779	+11.9	0.18
II	776	+8.9	0.10
III	724	−43.1	2.42
IV	778	+10.9	0.15
V(−)	771	+3.9	0.02
VI(+)	720	−47.1	2.89
VII	792	+24.9	0.81
VIII	759	−8.1	0.08
IX	823	+55.9	4.07
X	749	−18.1	0.43
			11.15
χ^2, $P=0.05=16.92$			

10. SEASONAL VARIATION IN THE AUTO- AND CROSS-CORRELATIONS

In section 4 the removal of the seasonal variation in the mean and variance was explained. Such a procedure does not, however, insure completely stationary series. Because the entire general circulation is more vigorous in the winter hemisphere, there is good reason to suspect that the latitudinal displacements and meanderings of the westerlies display different characteristics in the winter than in the summer season.

To check this possibility each of the 21 yr. of Z.I. data was divided into four seasons (December through February, etc.). For each year the first four lag correlation and cross-correlation coefficients were computed and these were then averaged over the 21 (or 20) yr. Table 4 summarizes the lag coefficients.

It is obvious from the table that a definite seasonal variation in the persistence structure of all three indices

TABLE 4.—*Seasonal autocorrelation coefficients*

Lag	Winter	Spring	Summer	Fall
Subtropical Z.I.				
1	0.89	0.87	0.84	0.81
2	0.74	0.72	0.66	0.57
3	0.62	0.59	0.51	0.38
4	0.53	0.51	0.41	0.27
Polar Z.I.				
1	0.85	0.87	0.86	0.80
2	0.64	0.66	0.64	0.52
3	0.50	0.48	0.46	0.33
4	0.39	0.34	0.32	0.21
Temperate Z.I.				
1	0.88	0.89	0.86	0.86
2	0.69	0.70	0.64	0.63
3	0.53	0.53	0.45	0.44
4	0.40	0.38	0.31	0.29

TABLE 5.—*Seasonal cross-correlation coefficients*

	Winter	Spring	Summer	Fall
Subtropical-Polar	−0.48	−0.48	−0.23	−0.20
Subtropical-Temperate	−0.68	−0.61	−0.29	−0.51
Polar-Temperate	−0.09	−0.15	−0.50	−0.41

exists. The greatest persistence is exhibited in the winter and spring and the least in the fall. The interpretation of this fact in the light of what is known about the general circulation is not entirely clear. Blocking, which has a strong seasonal maximum in spring and a minimum in fall (Rex [16]), could be responsible for the variation in persistence.

Table 5 gives the cross-correlation coefficient (lag 0) for the three combinations of Z.I. data. Again a strong seasonal effect is noticeable with the strongest negative relationship holding between the two indices spanning the belt of maximum westerlies. Thus, because of the annual contraction and expansion of the main belt of the westerlies, the subtropical and temperate indices are better correlated in the winter than in summer and vice versa for the polar-temperate combination. That a seasonal variation occurs in the relationship characteristic of an index cycle is suggested by the winter-spring maximum and fall minimum in the subtropical-polar cross correlation. This seems consistent with Namias and Clapp's preference for a "major index cycle" occurring most often in late winter.

The seasonal variation evident in the persistence structure of the zonal indices and their joint variation makes it clear that an additional source of nonstationarity exists which cannot be eliminated by standardizing for an annual variation in the mean and variance of each index. Such a nonstationarity complicates the interpretation of all the spectra and cross spectra. A subjective weighting of each of the figures 2–13 by the results of tables 4 and 5 is possible, so that, e.g., one could say that the polar-temperate cospectrum, figure 7, is strongly characteristic of the summer circulations and is weakly characteristic of winter circulations. It is not clear, however, that such a procedure is justified. Computation with sufficient resolution of reliable spectra and cross spectra for each season individually, a possible solution to the problem of nonstationarity, is not feasible with the length of record available.

11. CONCLUSIONS

Despite the limitations on the cross-spectral analysis imposed by the seasonal variation in the correlation structure of the Z.I. time series, some interesting results were produced. The cospectra of the Z.I. data exhibit a maximum negative covariance in a very broad range of period or frequency. This fact serves to reinforce the statement by Namias and Clapp [12], quoted in the Introduction, concerning the variability in length and intensity of the index cycle. From the coherence statistic no evidence is shown which indicates a phenomenon distinct from a broad continuum relationship among the Z.I. data.

It is tempting to speculate about the reasons why the 3- to 8-week-period range was selected by the authors cited in the Introduction. The possibility exists that the maximum covariability in the Z.I. data at various latitudes is perceived subjectively on a logarithmic (period) scale thus recognizing the minima appearing in the cospectra. Use of 5-day mean Z.I. data, as employed by the Extended Forecast Branch, would tend to produce an artificial "peaking" of the spectrum as would subjective elimination of the periods greater than 8 weeks made on the basis that these oscillations were seasonal trend.

However, if a phenomenon so distinct as to be named "the index cycle" has an existence apart from the shifts or displacements of the westerlies covering a very broad range of frequency characteristic of "normal" atmospheric variability, the coherence statistic, not the cross spectrum, should confirm that existence. The data do not give evidence for such a confirmation.

Certainly the results of this statistical analysis cannot be considered definitive. The zonal index data, calculated over fixed latitude bands, may not incorporate the critically characteristic features of an index cycle, thus obscuring its existence. Perhaps some combination of zonal and meridional indices would more nearly reflect the succession of circulation patterns thought to constitute an index cycle.

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